# MATHEMATICAL MODEL OF FUZZY CONTROL SYSTEM FOR AUTONOMOUS GUIDED VEHICLE IN 3D SPACE

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## Introduction

At present, the control loops of moving platforms are designed on the base of fuzzy control theory. Especially path searching in a 2D changing environment has received considerable attention as a part of the general problem of robot motion planning. A particularly interesting problem in this context is path planning with respect to a moving object. The design of such intelligent guided vehicles needs capabilities for environment recognition and motion planning.[1, 2, 3] Nowadays, fuzzy control is a promising technique for intelligent system design.[4, 5, 6] The most important feature of this method is that it eliminates the difference between goals and constraints and makes it possible to relate them in the decision-making process.[7] This paper presents a fuzzy control method for autonomous guided vehicle, which tracks an object in 3D space.

## Problem Statement

The problem involves finding the best possible time (optimal) registration path of the autonomous guided vehicle from the initial state to the goal. The goal is presented as a moving object, which the guided vehicle has to reach. It changes its coordinates during the decision-making process. The autonomous guided vehicle is modeled as a point, which movement is described by the following system of recurrent equations:

$$x_{k+1} = x_k + v.\cos\Psi_k.\sin\gamma_k.\Delta t \qquad (1)$$

$$y_{k+1} = y_k + v.\sin\Psi_k.\sin\gamma_k.\Delta t \qquad (2)$$

$$p_{k+1} = p_k + v_h.\Delta t \qquad (3)$$

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$$h_{k+1} = h_k + v_v.\Delta t \tag{4}$$

$$\Psi_{k+1} = \Psi_k + \Delta\Psi \tag{5}$$

$$\gamma_{k+1} = \gamma_k + \Delta\gamma \tag{6}$$

The states of the system are $x_k$, $y_k$, $p_k$, $h_k$,$\Psi_k$ and $\gamma_k$. The meaning of the notations is as follows:

- $x_k$, $y_k$, $h_k$ – coordinates of the model;
- $v_h = v.\sin\gamma_k$ – horizontal velocity;
- $v_v = v.\cos\gamma_k$ – vertical velocity;
- $v = \sqrt{v_h^2 + v_v^2}$ – total velocity;
- $p_k = \sqrt{x_k^2 + y_k^2}$ – total horizontal path;
- $\Psi_k$ – azimuth-path angle;
- $\gamma_k$ – flight-path angle;

The problem of control of the guided vehicle in 3D space is solved in horizontal and vertical planes. The first, second and the fifth equations are used in the horizontal plane. The azimuth-path angle $\Psi_k$ and the horizontal velocity $v_h$ control the vehicle. In the vertical plane the flight-path angle $\gamma_k$ and the velocity $v$ control it. In this case equations 3, 4, and 6 are used. The dynamic nature of the system can be modeled by constraining the angle of the velocity vector in the 3D space by its velocity $v$. The constraints are given in horizontal and vertical planes as follows:

In horizontal plane

$$\Psi = \begin{cases} \Psi_{max}, & \Psi \geq \Psi_{max} \\ \Psi, & -\Psi_{max} < \Psi < \Psi_{max} \\ -\Psi_{max}, & \Psi \leq -\Psi_{max} \end{cases}$$

In vertical plane

$$\gamma = \begin{cases} \gamma_{max}, & \gamma \geq \gamma_{max} \\ \gamma, & -\gamma_{max} < \gamma < \gamma_{max} \\ -\gamma_{max}, & \gamma \leq -\gamma_{max} \end{cases}$$

where:

$$\Psi_{max} = -55^\circ.\frac{v_h}{10}; \quad 55^\circ.\frac{v_h}{10}, \qquad \gamma_{max} = -30^\circ.\frac{v_v}{10}; \quad 30^\circ.\frac{v_v}{10}.$$

The problem is solved using dynamic programming in a fuzzy environment.[8, 9] The objective of the proposed algorithm is to demonstrate a fuzzy method for determination of the trajectory of the dynamic object, which is modeled as movement of a point in the 3D space. The following assumptions are made in the decision-making process:

– sets of alternatives: $X = \{x, y\}$, $P = \{p, h\}$;
– at each time $t$ the control azimuth-path angle $\Psi_t$ is subjected to a fuzzy constraint $C_t^{\Psi}$, which is a fuzzy set in $U$ characterized by a membership function in horizontal plane $\mu_t^{\Psi}(\Psi_t) = f(\Psi_t)$;
– at each time $t$ the control flight-path $\gamma_t$ is subjected to a fuzzy constraint $C_t^{\gamma}$, which is a fuzzy set in $U$ characterized by a membership function in vertical plane $\mu_t^{\gamma}(\Psi_t) = f(\gamma_t)$;
– the goal is a fuzzy set $G^N$ in $V$, which is characterized by two membership functions in horizontal and vertical planes. The fuzzy goal might be presented as two functions, which have maximums at the end points. Their combined influence may be respectively expressed in words as: “x, y, h *should be in the vicinity of* $x_g$, $y_g$, $h_g$.“ Their membership functions are given as follows:

$$\mu_G^h(x, y) = e^{-k_x (x_g - x)^2 - k_y (y_g - y)^2}, \qquad \mu_G^v(h) = e^{-k_p (p_g - p)^2 - k_h (h_g - h)^2}.$$

## Description of the Fuzzy Control

The problem of finding the optimal registration path of the model is a multi-stage decision process. The original multi-stage (N-stage) decision process is replaced with N one-stage processes. Dynamic programming is based on the principle of optimality. In the beginning the algorithm uses the reverse problem of dynamic programming. The model is moved from the goal to the initial state. The optimal movement trajectory is calculated in an off-line mode. The last step of the decision process is N for the reverse problem of dynamic programming. It is obtained when the model has attained the initial state. Then, the algorithm uses the direct problem of dynamic programming in an on-line mode.

*Problem:* The autonomous guided vehicle has to attain given goal, which is moving in given direction. The initial state $x_0$, $y_0$, $h_0$ and the coordinates $x_g$, $y_g$, $h_g$ of the goal are assumed to be given. The termination time of the process is N. The decision-making process has the following goal:

G: (x, y, h) should be in the vicinity of ($x_g$, $y_g$, $h_g$).

It has to find the best possible time registration path of the model from the initial state to the goal.

In the horizontal plane the initial and final conditions are defined as follows:

$$\mu_G^h(x_0, y_0) = \alpha; \qquad \mu_G^h(x_N, y_N) = 1;$$

The final condition for the control signal $\Psi_k$ is included implicitly. It has to determine the sequence:

$$\mu_G^h(x_{N-1}, y_{N-1}); \qquad \mu_{N-1}^{\Psi}(\Psi_{N-1});$$

$$\mu_G^h(x_{N-2}, y_{N-2}); \qquad \mu_{N-2}^{\Psi}(\Psi_{N-2});$$

$$\ldots\ldots\ldots\ldots \qquad \ldots\ldots\ldots$$

$$\mu_G^h(x_0, y_0); \qquad \mu_0^{\Psi}(\Psi_0);$$

which optimizes the criteria $\mu_G^h(x_k, y_k), \qquad k = 0,1,2,\ldots,N-1$.

In the vertical plane the initial and final conditions are defined as follows:

$$\mu_G^v(p_0, h_0) = \beta; \qquad \mu_G^v(p_N, h_N) = 1;$$

The final condition for the control signal $\gamma_k$ is included implicitly. It has to determine the sequence:

$$\mu_G^v(p_{N-1}, h_{N-1}); \qquad \mu_{N-1}^{\gamma}(\gamma_{N-1});$$

$$\mu_G^v(p_{N-2}, h_{N-2}); \qquad \mu_{N-1}^{\gamma}(\gamma_{N-2});$$

$$\ldots\ldots\ldots \qquad \ldots\ldots\ldots$$

$$\mu_G^v(p_0, h_0); \qquad \mu_0^{\gamma}(\gamma_0);.$$

which optimizes the criteria $\mu_G^v(p_k, h_k), \qquad k = 0,1,2,\dots,N-1$.

More explicitly, in terms of membership functions, the decision in the horizontal and vertical planes can be expressed as:

$$\mu_D^h(\Psi_0,\dots,\Psi_{N-1}) = \mu_0^\Psi(\Psi_0) \wedge \mu_1^\Psi(\Psi_1)\dots\mu_{N-1}^\Psi(\Psi_{N-1}) \wedge \mu_{G^N}^h(x_N, y_N);$$

$$\mu_D^v(\gamma_0,\dots,\gamma_{N-1}) = \mu_0^\gamma(\gamma_0) \wedge \mu_1^\gamma(\gamma_1)\dots\mu_{N-1}^\gamma(\gamma_{N-1}) \wedge \mu_{G^N}^v(p_N, h_N),$$

where:

- in the horizontal plane $x_N, y_N$ can be represented as a function of $x_0, y_0$ and $\Psi_0,\dots.\Psi_{N-1}$ through the iteration of equations 1 and 2;
- in the vertical plane $p_N, h_N$ can be represented as a function of $h_0$ and $\gamma_0,\dots.\gamma_{N-1}$ through the iteration of equations 3 and 4.

As is usually the case in multi-stage processes, it is convenient to express the solution in the form:

$$\Psi_t = \pi_t^h(x_t, y_t);$$

$$\gamma_t = \pi_t^v(p_t, h_t), \qquad t = 0,1,2,\dots,N-1,$$

where:

$\pi_t^h$ and $\pi_t^v$ are policy functions in the horizontal and vertical planes respectively. Then dynamic programming is applied to find the maximizing decisions $\Psi_0^M,\dots.\Psi_{N-1}^M$ and $\gamma_0^M,\dots\gamma_{N-1}^M$.

The following simplified expressions of equations 1, 2, 3, and 4 are used:

$$\left.\begin{aligned} x_{N-k+1} &= f(x_{N-k}, \Psi_{N-k}) \\ y_{N-k+1} &= f(y_{N-k}, \Psi_{N-k}) \end{aligned}\right\} \Rightarrow f(x_{N-k}, y_{N-k}, \Psi_{N-k}) \qquad (7)$$

$$\left.\begin{aligned} h_{N-k+1} &= f(h_{N-k},\gamma_{N-k}) \\ p_{N-k+1} &= f(p_{N-k},\gamma_{N-k}) \end{aligned}\right\} \Rightarrow f(p_{N-k},h_{N-k},\gamma_{N-k}) \tag{8}$$

More specifically, using a definition of the solution in a fuzzy environment [10] and equation (7), the solution in the horizontal plane can be written as:

$$\begin{aligned} \mu_D^h(\Psi_0^M,\ldots\Psi_{N-1}^M) = Max_{\Psi_0},\ldots Max_{\Psi_{N-2}}, Max_{\Psi_{N-1}}(\mu_0^{\Psi}(\Psi_0)\wedge\ldots\mu_{N-2}^{\Psi}(\Psi_{N-2}) \\ \wedge\,\mu_{N-1}^{\Psi}(\Psi_{N-1})\wedge\mu_{G^N}^h(f(x_{N-1},y_{N-1},\Psi_{N-1}))) \end{aligned} \tag{9}$$

In the same manner, using the approach of Bellman and Zadeh [11] and equation (8), the solution in the vertical plane can be written as:

$$\begin{aligned} \mu_D^v(\gamma_0^M,\ldots\gamma_{N-1}^M) = Max_{\gamma_0},\ldots Max_{\gamma_{N-2}}, Max_{\gamma_{N-1}}(\mu_0^{\gamma}(\gamma_0)\wedge\ldots\mu_{N-2}^{\gamma}(\gamma_{N-2}) \\ \wedge\,\mu_{N-1}^{\gamma}(\gamma_{N-1})\wedge\mu_{G^N}^v(f(p_{,N-1},h_{N-1},\gamma_{N-1}))) \end{aligned} \tag{10}$$

After some transformations expressions 9 and 10 can be rewritten in the following form:

$$\mu_{G^{N-1}}^h(x_{N-1},y_{N-1}) = Max_{\Psi_{N-1}}(\mu_{N-1}^{\Psi}(\Psi_{N-1})\wedge\mu_{G^N}^h(f(x_{N-1},y_{N-1},\Psi_{N-1}))) \tag{11}$$

$$\mu_{G^{N-1}}^v(p_{N-1},h_{N-1}) = Max_{\gamma_{N-1}}(\mu_{N-1}^{\gamma}(\gamma_{N-1})\wedge\mu_{G^N}^v(f(p_{N-1},h_{N-1},\gamma_{N-1}))) \tag{12}$$

Equations 11 and 12 may be regarded as the membership functions of a fuzzy goal at time $t = N-1$, which is induced by the given goal $G^N$ at time $t = N$.

Repeating this backward iteration, which is a simple instance of dynamic programming, the following recurrence equations are obtained:

$$\mu_{G^{N-k}}^h(x_{N-k},y_{N-k}) = Max_{\Psi_{N-k}}(\mu_{N-k}^{\Psi}(\Psi_{N-k})\wedge\mu_{G^{N-k+1}}^h(x_{N-k+1},y_{N-k+1})) \tag{13}$$

$$\mu_{G^{N-k}}^v(p_{N-k},h_{N-k}) = Max_{\gamma_{N-k}}(\mu_{N-k}^{\gamma}(\gamma_{N-k})\wedge\mu_{G^{N-k+1}}^v(p_{N-k+1},h_{N-k+1})) \tag{14}$$

In equations (13) and (14) unknowns are the membership functions of the control values $\mu_t^{\Psi}(\Psi_t)$ and $\mu_t^{\gamma}(\gamma_t)$. The memberships functions of the goal are calculated as follows:

$$\mu_{G^{N-k}}^{h}\left(x_{N-k}, y_{N-k}\right) = e^{-k_x\left(x_g - x_{N-k}\right)^2 - k_y\left(y_g - y_{N-k}\right)^2} = A_{N-k} \tag{15}$$

$$\mu_{G^{N-k}}^{v}\left(p_{N-k}, h_{N-k}\right) = e^{-k_p\left(p_g - p_{N-k}\right)^2 - k_h\left(h_g - h_{N-k}\right)^2} = B_{N-k} \tag{16}$$

where $A_{N-k}$ and $B_{N-k}$ are the calculated values of the membership function at moment $N-k$.

To determine the control signals $\Psi_{N-k}$ and $\gamma_{N-k}$ at $N-k$, one has to represent the system equations 1, 2, 3, and 4 as follows:

$$x_{N-k} = x_{N-k+1} - v.\cos(\Psi_{N-k}).\sin(\gamma_{N-k}).\Delta t \tag{17}$$

$$y_{N-k} = y_{N-k+1} - v.\sin(\Psi_{N-k}).\sin(\gamma_{N-k}).\Delta t \tag{18}$$

$$p_{N-k} = p_{N-k+1} - v.\sin(\gamma_{N-k}).\Delta t \tag{19}$$

$$h_{N-k} = h_{N-k+1} - v.\cos(\gamma_{N-k}).\Delta t \tag{20}$$

The procedure for finding the control angles $\Psi_0^M, \ldots, \Psi_{N-1}^M$ and $\gamma_0^M, \ldots, \gamma_{N-1}^M$ is presented below:

a. The goal coordinates are initialized $x_{N-k} = x_g$, $y_{N-k} = y_g$, $p_{N-k} = \sqrt{x_g^2 + y_g^2}$, $h_{N-k} = h_g$, initials values for $\Psi_{N-k}$, $\gamma_{N-k}$ and $k = N$ are also given;

b. $\Psi_{N-k} = 0$;

c. $\mu_{G^{N-k}}^{h}(x_{N-k}, y_{N-k})$ is calculated using equation (13);

d. Equation (15) is computed, where $x_{N-k}$ and $y_{N-k}$ are replaced with expressions 17 and 18;

e. $\Psi_{N-k} = \Psi_{N-k} + \Delta\Psi$ and points *c*, *d*, *e* are repeated until the maximum of $\Psi_{N-k}^M$ is found;

f. $\gamma_{N-k} = 0$;

g. $\mu^{v}_{G^{N-k}}(p_{N-k}, h_{N-k})$ is calculated using expression 14, where $x_{N-k}$ and $y_{N-k}$ are the values attained at the maximum $\Psi_{N-k} = \Psi^{M}_{N-k}$;

h. Equation 16 is computed, where $p_{N-k}$ and $h_{N-k}$ are replaced with expressions 19 and 20 respectively;

i. $\gamma_{N-k} = \gamma_{N-k} + \Delta\gamma$ and points *g, h, i* are repeated while the maximum of $\gamma^{M}_{N-k}$ is found;

j. If $(x_g \neq x_0 \; or \; y_g \neq y_0 \; or \; h_g \neq h_0)$ then

$x_{N-k}$, $y_{N-k}$, $p_{N-k}$ and $h_{N-k}$ are calculated using expressions 17, 18, 19, and 20;

the horizontal velocity $v_h = \sqrt{v^2 - \left(\frac{h_{N-k} - h_{N-k+1}}{\Delta t}\right)^2}$ is computed;

$k = k - 1$;

go to point b;

else

end of procedure;

In the process of searching for the maximizing decision the two control angles are altered from 0 to 2π for each of the one-stage decision process. The increments of the control angles ΔΨ and Δγ are calculated by the following expressions:

$\Delta\Psi = \frac{2.\pi}{hwin}$, where *hwin* is the width of the matching window in the horizontal plane,

and

$\Delta\gamma = \frac{2.\pi}{vwin}$, where *vwin* is the width of the matching window in the vertical plane, respectively.

**Simulation**

In order to explore the validity of the proposed fuzzy control for autonomous guided vehicle simulations have been carried out. The following initial conditions have been given: $v$=6.0, $\Delta$t=1.0, $k_x$=0.00001, $k_y$=0.00001, $k_h$=0.0001, $k_p$=0.00001, hwnd=720, vwnd=720, $x_g$=500, $y_g$=320, $h_g$=200, $x_0$=10, $y_0$=180, $h_0$=50. The process starts when k reaches 120. The simulation program is written in MATLAB. The moving goal is described by the following linear movement equations:

$x_0$=$x_0$+2;  $y_0$=$y_0$+4;  $h_0$=$h_0$+2;  $p_0$=sqrt(($x_0$.^2)+($y_0$.^2));

Figure 1 shows the simulation results in 3D space. Figure 2 shows the horizontal and vertical velocity in the process of decision-making.

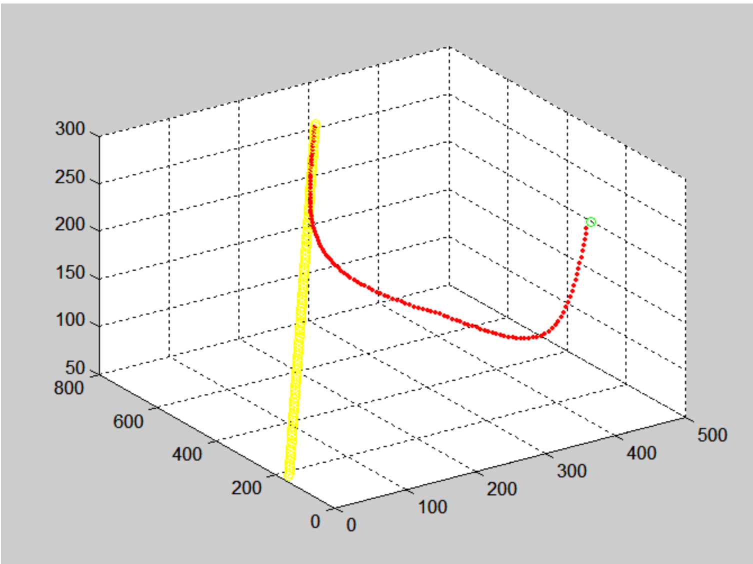


Figure 1: Moving trajectories of the goal and the vehicle in 3D space

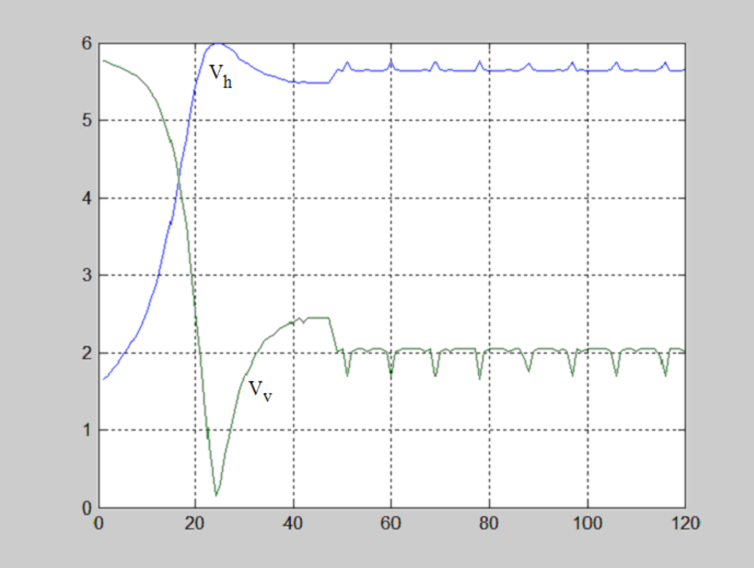


Figure 2: Horizontal and vetical velocity.

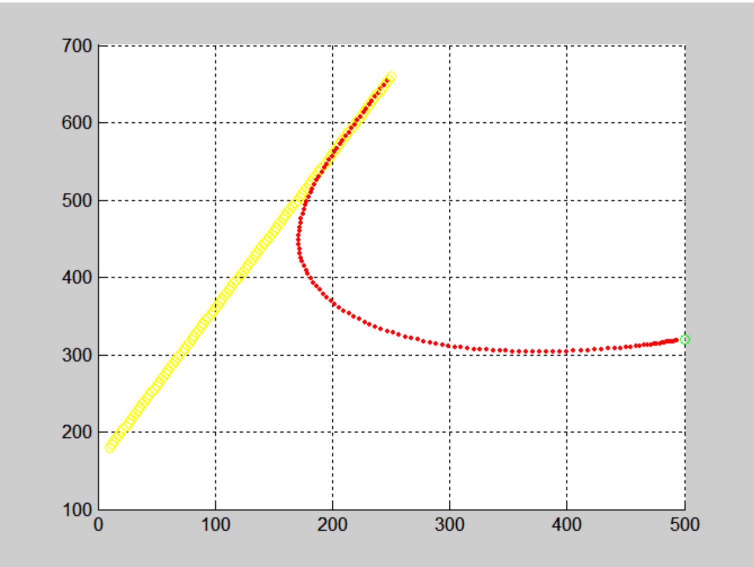


Figure 3: Moving trajectories in horizontal plane.

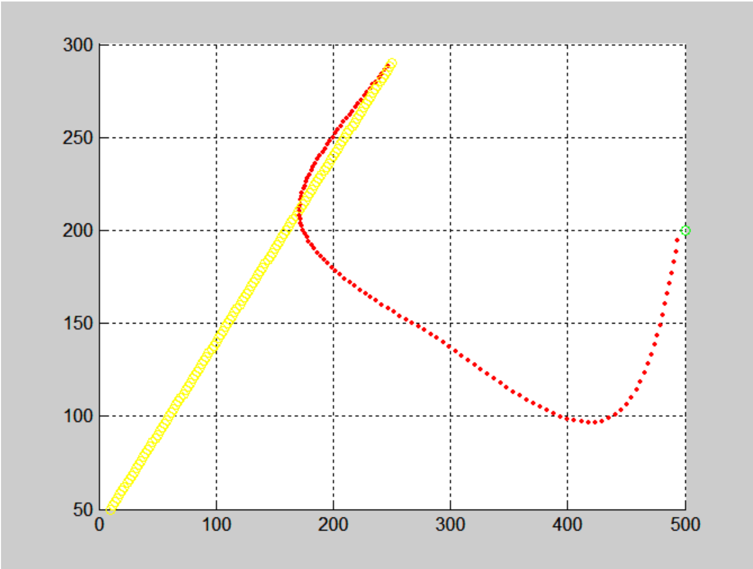


Figure 4: Moving trajectories in vertical plane.

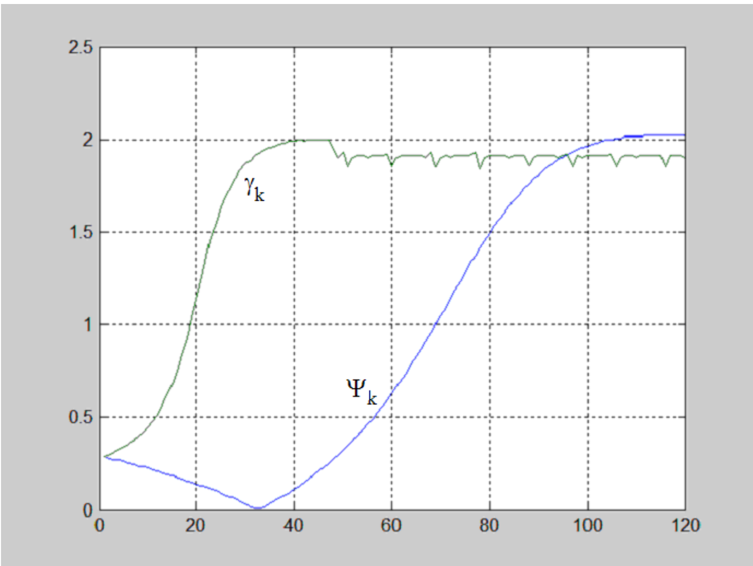


Figure 5: Control angles in horizontal and vertical planes.

Figure 3 and Figure 4 show the simulation results in horizontal and vertical planes. The control angles are shown in Figure 5. It is observed that the guided vehicle can reach the moving object successfully.

## Conclusion

A fuzzy control approach has been presented for model movement along the optimal path. The optimal registration path is computed for an autonomous guided vehicle in 3D space. A system of functional equations can be solved using dynamic programming and appropriate membership functions for fuzzy environment. In this context, the proposed approach can be used for other similar applications. Computer simulations have validated the validity of the proposed method for control. Applications. Experiments of the guided vehicle are simulated in a simple fuzzy environment. In addition, techniques to plan a path in an expanded fuzzy environment, including both stationary and moving obstacles, are under study.

## Notes:

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