

INTERACTING MULTIPLE MODEL ALGORITHM FOR MANOEUVRING SHIP TRACKING BASED ON NEW SHIP MODELS

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1. Introduction

Tracking of manoeuvring targets is a problem of a great practical and theoretical interest. The real-world tracking applications meet a number of difficulties caused by the presence of different kinds of uncertainty due to the unknown or not precisely known system model and random processes' statistics or because of their abrupt changes.^{2-5, 9} These problems are especially complicated in the marine navigation practice,^{7, 14, 15, 19} where the commonly used simple models of rectilinear or curvilinear target motions do not match the highly non-linear dynamics of the manoeuvring ship. A solution of these problems is to derive more adequate descriptions of the real ship dynamics and to design adaptive estimation algorithms.

Such a solution is proposed in the paper. A new ship model is derived in Section 2 after an analysis of the basic hydrodynamic models. The derived model is implemented in a new version of the Interacting Multiple Model (IMM) tracking algorithm - the most cost-effective multiple model algorithm for hybrid estimation.^{3, 6, 10, 11} The proposed model and tracking algorithm are presented in Section 3 and evaluated in Section 4.

2. Model identification

Results of a study, described in^{16, 17, 18} are summarised in this section. It should be noted that the high complexity of the hydrodynamic processes caused by the ship motion in deep and confined water and the wide variety of ship forms and sizes lead to various non-stochastic ship models. These models can be divided in two groups: precise models, topical for particular ship forms and sizes (the Sobolev,¹⁹ Cubic,¹ Quadratic¹³ and MMG¹⁴ models) and models with greater generality but lower

accuracy (the Pershitz¹⁵ and Nomoto¹² models). Here, the widely used continuous-time (CT) Pershitz model¹⁵ is chosen as a basic model to assure a good trade-off between complexity and accuracy:

$$\frac{dX}{dt} = K_V V_0 \sin(\psi - \beta), \quad (1)$$

$$\frac{dY}{dt} = K_V V_0 \cos(\psi - \beta), \quad (2)$$

$$\frac{d\psi}{dt} = K_V \omega, \quad (3)$$

$$\frac{d\omega}{dt} = -\left(\frac{V_0}{L}\right)^2 (q_{31}\beta + s_{31}\delta) - \frac{V_0}{L} r_{31}\omega, \quad (4)$$

$$\frac{d\beta}{dt} = -\frac{V_0}{L} (q_{21}\beta + h_1\beta|\beta| + s_{21}\delta) - r_{21}\omega, \quad (5)$$

$$V = V_0 K_V; \quad (6)$$

$$K_V = \frac{V(\omega)}{V(0)} = \frac{V}{V_0} = \left(1 + 1.9\omega^2 L^2 V_0^{-2}\right)^{-1} \leq 1.$$

The state vector of the considered CT model is $x = [X, Y, \psi, \omega, \beta, V]^T$. It includes the ship coordinates and heading, rate of turn, drift angle and velocity; δ is the control rudder angle deviation. The constants q_{21} , r_{21} , s_{21} , h_1 , q_{31} , r_{31} and s_{31} are hydrodynamic coefficients depending on the ship geometry, most of all, and on the ship length L .²⁰ Equations (3) and (6) illustrate the main feature of the considered dynamics - *the non-linear dependence between the rate of turn and the velocity of the ship*. This is the main difference between the presented model in this paper and other well-known simple models.^{2, 5, 9}

Very often in the available literature sources^{15, 20} this model is simplified by substituting the factor $|\beta|$ with an off-line computed factor:

$$\beta_0 = \frac{-q + \sqrt{q^2 + 4h_1 r_{31} s |\delta|}}{2h_1 r_{31}},$$

where: $q = q_{21}r_{31} - q_{31}r_{21}$, $s = r_{21}s_{31} - r_{31}s_{21}$. Then, the system of two first-order differential equations consisting of equation (4) and the modified equation (5) is transformed in two independent second-order differential equations, omitting the negligible second-order derivatives:

$$2p \frac{d\omega}{dt} \frac{L^2}{V_0^2} + q\omega \frac{L}{V_0} + s_{31}\delta = 0; \quad (4')$$

$$2p \frac{d\beta}{dt} \frac{L}{V_0} + q\beta + s_{21}\delta = 0, \quad (5')$$

where $p = 0.5(q_{21}^* + r_{31})$, $q^* = q_{21}^*r_{31} - q_{31}r_{21}$, $q_{21}^* = q_{21} + h_1\beta_0$. The final CT model (1)-(3), (4') and (6) is obtained by setting $\beta \equiv 0$.

The respective discrete-time (DT) model is:

$$X_{k+1} = X_k + TV_k \sin \psi_k, \quad (7)$$

$$Y_{k+1} = Y_k + TV_k \cos \psi_k, \quad (8)$$

$$\psi_{k+1} = \psi_k + TV_k \left[\Omega_k + 0.5T\tau V_k (\Omega_k - \Omega_0) e^{TV_k\tau} \right], \quad (9)$$

$$\Omega_{k+1} = \Omega_k e^{TV_k\tau} + \Omega_0 (1 - e^{TV_k\tau}), \quad (10)$$

$$V_k = V_0 K_V = V_0 (1 + 1.9\Omega_k^2 L^2)^{-1}, \quad (11)$$

where

$$\tau = \frac{-0.5p + \sqrt{0.25p^2 - q^*}}{L}, \quad [m^{-1}],$$

$$\Omega_0 = \frac{\omega}{V_0} = -\frac{[s_{31}\delta + \text{sign}(\delta)q_{31}\beta_0]}{r_{31}L}, \quad \left[\frac{\text{rad}}{m} \right].$$

and $k = 1, 2, \dots$; T is the sampling interval.

The full coincidence between the results obtained by the CT model (1)-(6), and these obtained by the derived DT model (7)-(11) is demonstrated in ¹⁷. Model (7)-(11) is used for true data generation in further simulations.

The final DT model, suitable for implementation in Kalman filter, is composed on the basis of the assumptions^{17, 18}:

- * It is assumed that the observed ship maneuvers with a constant rate of turn:

$$\Omega_{k+1} = \Omega_k \quad (\text{i.e. } \tau \equiv 0).$$

- * The whole domain of unknown control parameters Ω_k is replaced by a set of three control parameters corresponding to the three basic kinds of ship motions: rectilinear motion (Ω_1), left and right turns (Ω_2 and Ω_3):

$$\Omega = [\Omega_1, \Omega_2, \Omega_3]^T = [0, U, -U]^T,$$

where U denotes the preset constant rate of turn. The vector Ω covers all possible ship manoeuvres and system noises in the band $[U, -U]$. The particular choice of U is made by taking into account general considerations from the marine practice and some important international navigation restrictions.²⁰

- * The attempt to introduce respective vector of possible ship lengths has been recognised in ¹⁷ as unsuccessful because of bad distinction of the resulting models. The uncertainty, concerning the ship geometry has been overcome by introducing a constant average ship length $l = \text{const}$.¹⁷

So, the final version of the requested ship model takes the following general form:

$$x_{i,k+1} = f(x_k, \Omega_i, l), \quad i = 1, 2, 3,$$

where $x_k = [X_k, Y_k, \psi_k, V_{0,k}]^T$ and where:

$$X_{i,k+1} = X_{i,k} + TV_{i,k+1} \sin \psi_{i,k}, \quad (12)$$

$$Y_{i,k+1} = Y_{i,k} + TV_{i,k+1} \cos \psi_{i,k}, \quad (13)$$

$$\psi_{i,k+1} = \psi_{i,k} + TV_{i,k+1} \Omega_i, \quad (14)$$

$$V_{i,k+1} = K_{V,i} V_{0,k}, \quad (15)$$

where

$$K_{V,i} = \left(1 + 1.9\Omega_i^2 l^2\right)^{-1}, \quad \Omega = [\Omega_1, \Omega_2, \Omega_3]' = [0, U, -U]'$$

Another model, based on the extended state vector $x_{i,k}^e = [X_{i,k}, Y_{i,k}, \psi_{i,k}, V_{i,k}, \Delta\Omega_{i,k}]'$ is suggested in ¹⁸. The corresponding extended ship models ($i = 1, 2, 3$) have the form:

$$X_{i,k+1} = X_{i,k} + TV_{i,k+1} \sin \psi_{i,k}, \quad (16)$$

$$Y_{i,k+1} = Y_{i,k} + TV_{i,k+1} \cos \psi_{i,k}, \quad (17)$$

$$\psi_{i,k+1} = \psi_{i,k} + TV_{i,k+1} (\Omega_i + \Delta\Omega_{i,k}), \quad (18)$$

$$V_{i,k+1} = K_{V,i} V_{0,k}, \quad (19)$$

$$\Delta\Omega_{i,k+1} = \Delta\Omega_{i,k}, \quad (20)$$

where $K_{V,i} = \left(1 + 1.9\Omega_i^2 l^2\right)^{-1}$. It takes into account possible differences $\Delta\Omega_{i,k}$ between the unknown true value of the ship rate of turn Ω_k and its values Ω_i fixed in the IMM algorithm. The influence of $\Delta\Omega_{i,k}$ on the velocity is not taken into account because of its insignificance.

3. IMM algorithm for tracking of manoeuvring ship

Models (12)-(15) and (16)-(20) are expanded in ¹⁷ in Taylor time-series up to first-order terms around the estimated state vector. They are used in an Extended Kalman Filters (EKF) and respective IMM algorithms. The IMM algorithm based on model (12)-(15) is denoted as IMM-A and the proposed IMM algorithm based on model (16)-(20) is denoted as IMM-B.

The measurement equation has the form:

$$y_k = Hx_k + w_k,$$

where H is the measurement matrix,

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

w_k is a white Gaussian measurement noise with covariance matrix R_k .

For convenience, the polar measurements “range-bearing” $y_k = [r_k, \beta_k]^t$, are transformed here in Cartesian ones:

$$X_k = r_k \sin \beta_k, Y_k = r_k \cos \beta_k.$$

So, the measurement vector acquires the new form $y_k = [X_k, Y_k]^t$. Respectively, the covariance matrix of the measurement errors is⁸:

$$R_{i,k} = \begin{bmatrix} \sigma_r^2 \sin^2 \beta_k + r_k^2 \sigma_\beta^2 \cos^2 \beta_k & (\sigma_r^2 - r_k^2 \sigma_\beta^2) \sin \beta_k \cos \beta_k \\ (\sigma_r^2 - r_k^2 \sigma_\beta^2) \sin \beta_k \cos \beta_k & \sigma_r^2 \cos^2 \beta_k + r_k^2 \sigma_\beta^2 \sin^2 \beta_k \end{bmatrix},$$

where σ_r and σ_β are the standard deviations of the range and bearing angle.

The equations of the i^{th} ($i = 1, 2, 3$) EKF are:

$$\hat{x}_{i,k/k} = \hat{x}_{i,k/k-1} + K_{i,k} \gamma_{i,k}$$

$$\hat{x}_{i,k+1/k} = f(\hat{x}_{i,k/k}, \Omega_i, l),$$

$$\gamma_{i,k} = y_k - H_i \hat{x}_{i,k/k-1},$$

$$P_{i,k/k-1} = \phi f_i^x P_{i,k-1/k-1} (f_i^x)',$$

$$S_{i,k} = H_i P_{i,k/k-1} H_i' + R_{i,k},$$

$$K_{i,k} = P_{i,k/k-1} H_i' S_{i,k}^{-1},$$

$$P_{i,k/k} = P_{i,k/k-1} - K_{i,k} S_{i,k} K_{i,k}'.$$

Here, $\hat{x}_{i,k/k}$ and $\hat{x}_{i,k+1/k}$ are the filtered estimate of the state x_k and its one-step prediction; $\gamma_{i,k}$ and $S_{i,k}$ are the filter residual process and its covariance matrix, $P_{i,k/k}$ is the error covariance matrix, $K_{i,k}$ is the filter gain matrix, $\phi \geq 1$ is the fudge factor.

The Jacobi matrix $f_i^x = \left. \frac{\partial f_i}{\partial x_i} \right|_{x_i = \hat{x}_i}$ computed based upon the model (12)-(15) has the form:

$$f_i^x = \begin{bmatrix} 1 & 0 & TV_{i,k+1} \cos \hat{\psi}_{i,k} & TK_{V,i} \sin \hat{\psi}_{i,k} \\ 0 & 1 & -TV_{i,k+1} \sin \hat{\psi}_{i,k} & TK_{V,i} \cos \hat{\psi}_{i,k} \\ 0 & 0 & 1 & TK_{V,i} \Omega_i \\ 0 & 0 & 0 & K_{V,i} \end{bmatrix};$$

the one based on model (16)-(20) is:

$$f_i^x = \begin{bmatrix} 1 & 0 & TV_{i,k+1} \cos \hat{\psi}_{i,k} & TK_{V,i} \sin \hat{\psi}_{i,k} & 0 \\ 0 & 1 & -TV_{i,k+1} \sin \hat{\psi}_{i,k} & TK_{V,i} \cos \hat{\psi}_{i,k} & 0 \\ 0 & 0 & 1 & TK_{V,i} (\Omega_i + \Delta \Omega_{i,k}) & TV_{i,k+1} \\ 0 & 0 & 0 & K_{V,i} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

A hard logic is introduced in both IMM algorithms to avoid undesired combination of the estimates $\hat{V}_{1,k}$, $\hat{V}_{2,k}$ and $\hat{V}_{3,k}$ ¹⁷:

$$\hat{V}_{i,k} = \hat{V}_{1,k}, \quad (i = 2,3);$$

$$\hat{V}_k = \hat{V}_{1,k}, \quad \text{if } \mu_{1,k} > 0.5,$$

where $\mu_{i,k}$ is the probability of the event: “the i^{th} model is topical at time k ”, \hat{V}_k is the overall estimate of the ship velocity.

4. Performance evaluation

The performance of both IMM algorithms is compared by Monte Carlo simulations.² Results for 100 [independent](#) runs, each one lasting 200 scans (600s, $T=3s$) are given.

The simulation parameters of the true model (7)-(11) are standard^{20,17}: $q_{21} = 0.331$, $r_{21} = -0.629$, $s_{21} = -0.104$, $h_1 = 3.5$, $q_{31} = -4.64$, $r_{31} = 3.88$, $s_{31} = -1.019$, $L=99m$, $\delta_{\min} = 3^\circ$, $\delta_{\max} = 30^\circ$. The chosen initial conditions are: $X_0 = Y_0 = 10000m$, $\Psi_0 = 45^\circ$, $V_0 = 30 m/s$. Initially the ship moves rectilinearly. The applied pulse-wise rudder angle control law is:

$$\delta = \begin{cases} \delta_{\max}, & k \in [51, 67] \\ 0, & k \notin [51, 67] \end{cases}.$$

The true ship trajectory is presented in Fig.1.

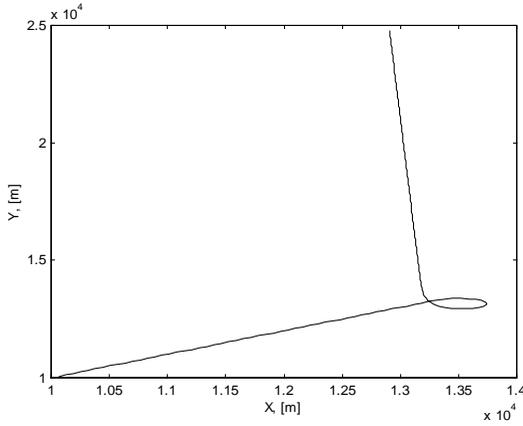


Figure 1: The true ship trajectory

Both considered IMM algorithms use constant ship length $l=69\text{ m}$ for each of the three models, control parameter $U=0.0066\text{ m}^{-1}$ ($360^\circ/\text{min}$) and fudge factors $\phi^A = \phi^B = 1.03$. To compute the measurement error covariance matrix, it is preset: $\sigma_r = 100\text{ m}$, $\sigma_\beta = 0.3^\circ$. The initial error covariance matrices $P_{i,0}$, the initial mode probability vectors μ and the transition probability matrices Pr are chosen as follows:

$$P_{i,0}^A = \text{diag}\{\sigma_X^2 \quad \sigma_Y^2 \quad \sigma_\psi^2 \quad \sigma_V^2\}, \quad P_{i,0}^B = \text{diag}\{\sigma_X^2 \quad \sigma_Y^2 \quad \sigma_\psi^2 \quad \sigma_V^2 \quad \sigma_{\Delta\Omega}^2\},$$

$$\mu^A = \mu^B = \begin{bmatrix} 0.95 \\ 0.025 \\ 0.025 \end{bmatrix}, \quad Pr^A = Pr^B = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.5 & 0.5 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}.$$

$$\sigma_X = \sigma_Y = \sigma_r, \quad \sigma_\psi = 0.1^\circ, \quad \sigma_V = 10\text{ m}, \quad \sigma_{\Delta\Omega} = 0.01\text{ rad/m}.$$

The Monte Carlo simulation results are shown in Figs. 2-13. General estimation of the algorithms' performance is given in Fig.2. The IMM-B algorithm possesses better consistency during the manoeuvring stage.

These inferences are confirmed by results received for the *mean error (ME)* and the *root mean square errors (RMSE)* of the state vector² (Figs.3-6 and Figs.7-10). The

average mode probabilities are presented in Figs.11-12. The computed ME of the estimated IMM-B control parameter change is given in Fig.13.

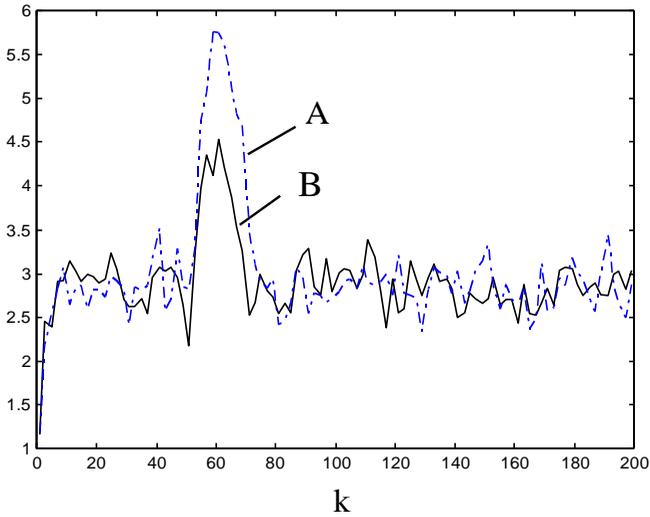


Figure 2: Normalized Estimation Error Squared

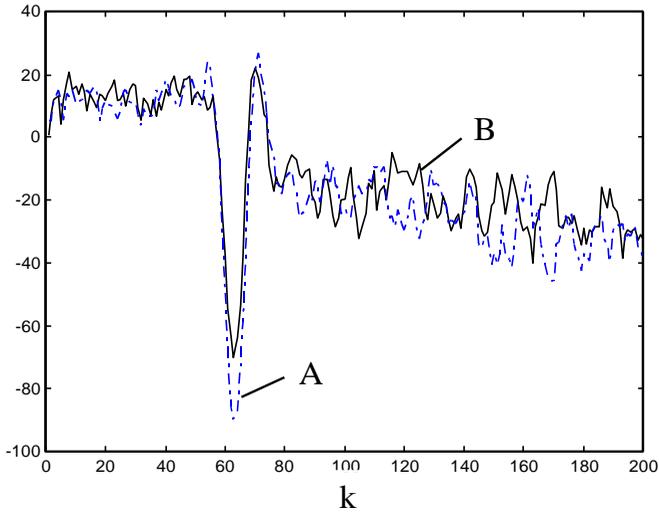


Figure 3: X Position ME

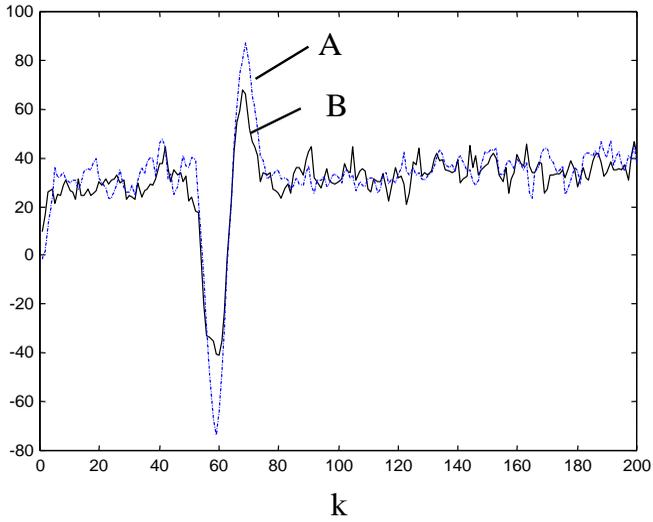


Figure 4: Y Position ME

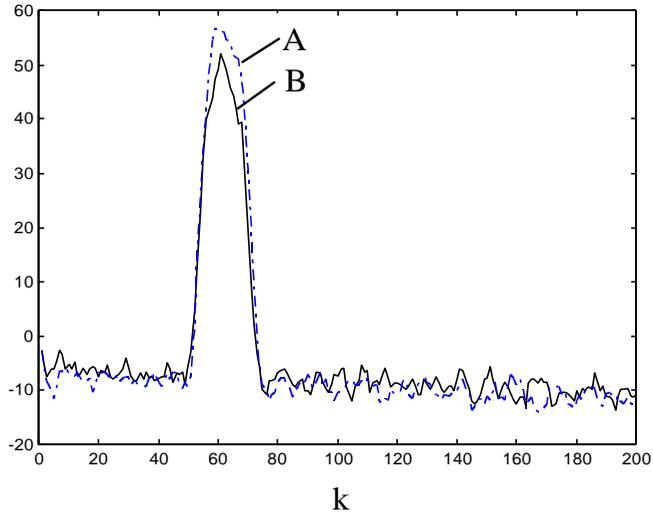


Figure 5: Heading ME

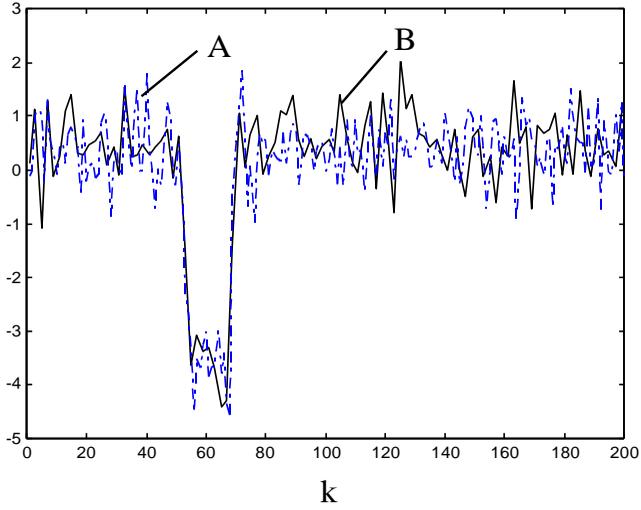


Figure 6: Velocity ME

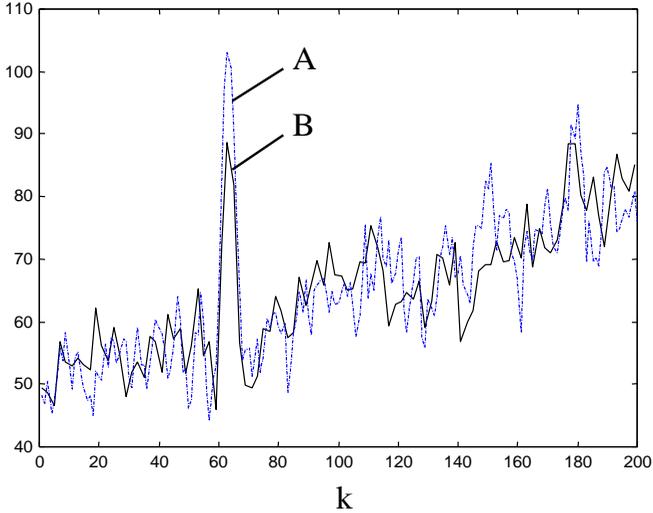


Figure 7: X Position RMSE

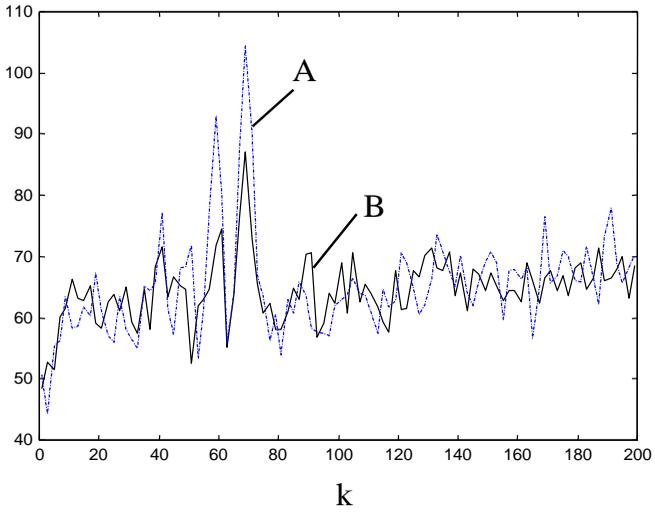


Figure 8: Y Position RMSE

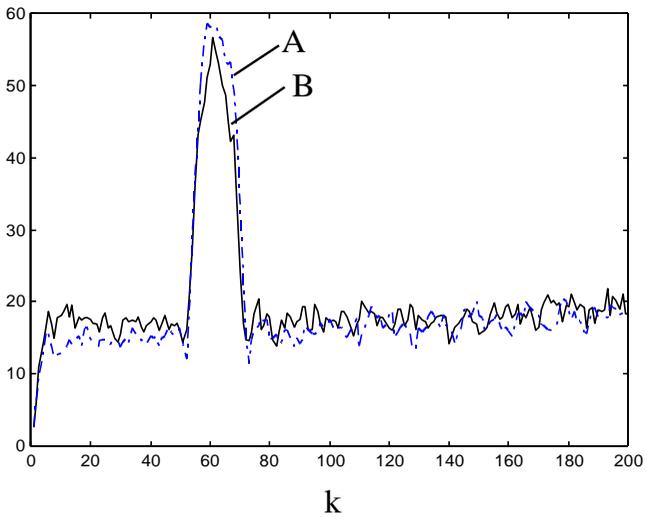


Figure 9: Heading RMSE

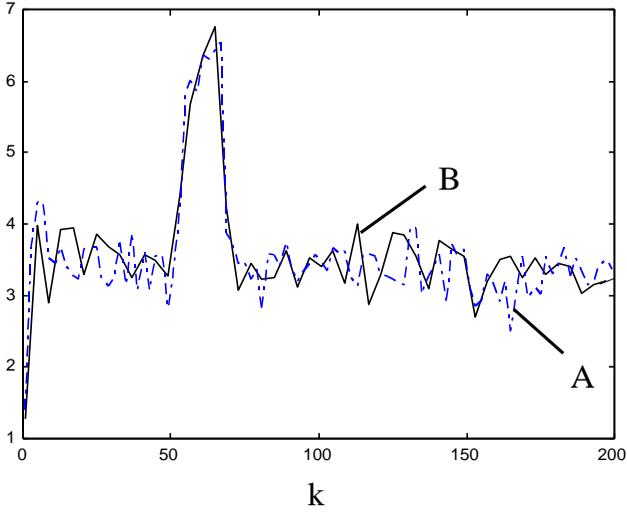


Figure 10: Velocity RMSE

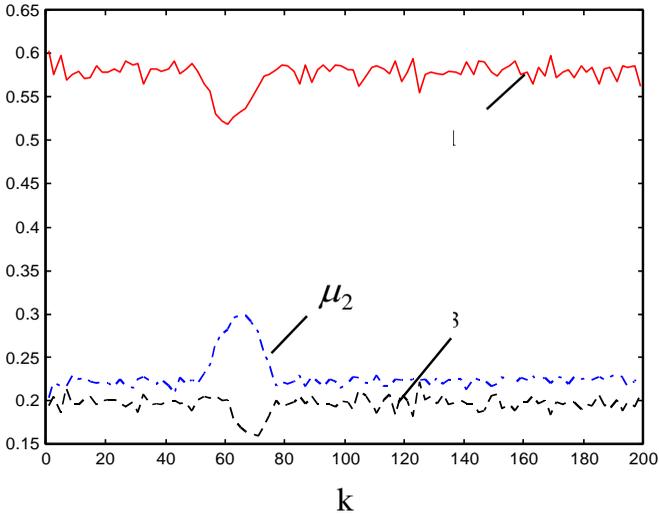


Figure 11: Average Mode Probabilities for IMM-A

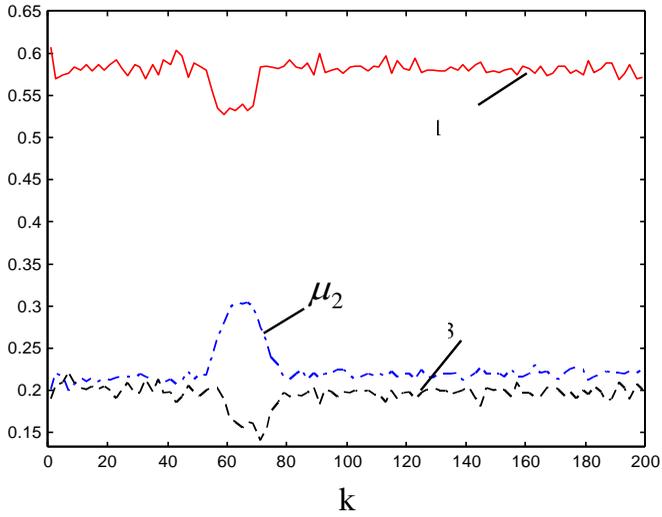


Figure 12: Average Mode Probabilities for IMM-B

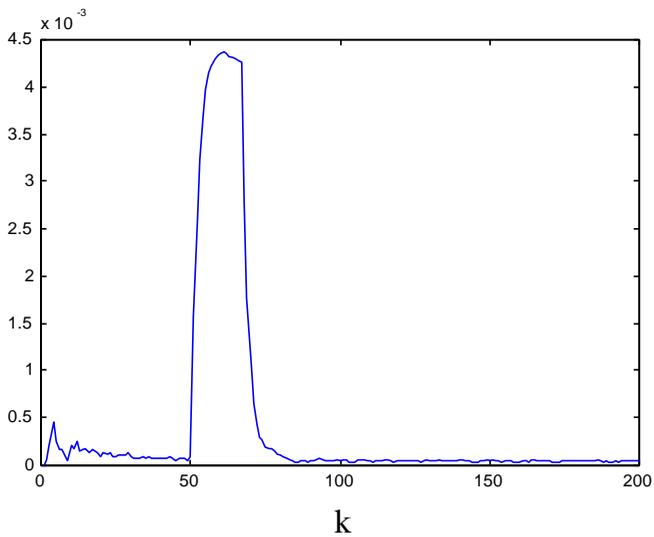


Figure 13: IMM-B control parameter change ME

5. Conclusions

New models adequately describing the non-linear dynamics of maneuvering ship motion are proposed in the paper for manoeuvring ship tracking. They are implemented in a standard and in newly designed IMM versions. The proposed new IMM uses extended state vector and model to compensate the difference between the fixed control parameter of the currently used IMM model and its real value. The performed Monte Carlo simulations show excellent model fit and estimation performance.

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