MULTIPLE HYPOTHESIS TRACKING USING HOUGH TRANSFORM TRACK DETECTOR

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1. Introduction

The Multiple Hypothesis Tracking algorithm (MHT) is an effective algorithm for moving objects detection and tracking.[1,2] Few versions of this complex algorithm are described and evaluated in [1,2,4]. Its *measurement oriented* version is considered as the most effective from theoretical point of view, but its practical implementation is limited because of the required significant computational load in cluttered environment. Several techniques minimizing this load were proposed,[1,2,4] but they do not provide general solution to these problems. A new problem solution is proposed in this paper. A Hough Transform (HT) track detector is used for preliminary filtering of arriving false alarms (FA). The tracks detected in this way are processed asynchronously with another standard MHT algorithm to include them in the overall MHT scheme. The standard and the proposed MHT-HT algorithm (MHT^2-HT) are evaluated and compared in the paper. The proposed algorithm shows remarkably good performance in cluttered environment at the cost of delayed track detection process.

2. HT track detector

The Hough transform algorithm (HTA) maps each point from feature space (FS, or the space of measurements) to a curve in parameter space (PS).[3,5] If a set of points in FS lies along a straight line, the corresponding curves intersect in a single point in PS. An appropriate mapping equation is proposed[5]:

$$\rho = r \sin(\theta - \alpha), \tag{1}$$

where (r,α) is 2-D measurement vector in FS; ρ and θ are trajectory shift and heading. The range $r \in [0,r_{max}]$ and the azimuth $\alpha \in [0,360^o]$) arrive in radar polar coordinate system $rO\alpha$ oriented to the "North" direction.

In the HTA the trajectory is searched among a fixed finite set of $N_\theta N_\rho$ trajectories, with the following standard headings and shifts: $\theta_l = l\,\delta\theta \in \left[0,360^o\right]$, $l = \overline{0,N_\theta}$ and $\rho_m = m\delta\rho \in \left[0,\rho_{max}\right]$, $m = \overline{0,N_\rho}$; $\delta\theta$ and $\delta\rho$ are the primary discretization steps of the PS. For each measurement (r,α) HTA consecutively substitutes increasing values of θ_l to compute the shifts $\rho_l = \rho(r,\alpha,\theta_l)$, $l = \overline{0,N_\theta}$ - the addresses of measurement votes.

If the discrete heading coincides with the real one ($\theta_l = \theta$), the peak of votes will locate the parameters of the real trajectory (Fig. 1).

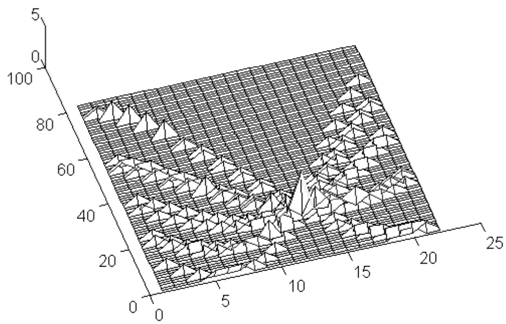

Figure 1: Peak location

If the real trajectory does not coincide with any standard one or, if there are measurement errors, the HTA detects (with some probability) the standard trajectory with the closest shift and heading instead the real one.

Appropriate equations determine the HTA accumulator size ($\Delta\rho, \Delta\theta$) and PS discretization steps as functions of sensor's measurement errors $\delta r \sim N\left(0,\sigma_r\right)$ and $\delta\alpha \sim N\left(0,\sigma_\alpha\right)$ at given probability of successful measurement vote P_G.[5] They define strips in FS, which shape cover the area of spatial measurement oscillations around known trajectory $\left(\rho,\theta\right)$ with a desired probability P_G. The false alarms accumulation and the reduction of HTA's resolving abilities are avoided in this way.

The PS discretization steps $\left(\delta\rho,\delta\theta\right)$ determine the worst case of non-coincidence between standard HT trajectories and an arbitrary chosen one $\left(\rho,\theta\right)$. The closest accumulator has coordinates:

$$\theta_{cl} = \delta\theta\, round\left(\frac{\theta}{\delta\theta}\right),\ \rho_{cl} = \delta\rho\, round\left(\frac{\rho}{\delta\rho}\right).$$

The strip shape is formed as a sum of n rectangular sub-strips (corresponding to sub-accumulators $(\rho_j, \theta_i), i = \overline{i_1, i_n}$) with size $(\Delta\rho, \delta\theta)$, where $\Delta\theta = n\delta\theta$.

The probability P_{G^*} of the event *'measurement hits a strip'* is a product of probabilities corresponding to the independent events: '$|\delta r| \le k\sigma_r$' and '$|\delta\alpha| \le k\sigma_\alpha$':

$$P_{G^*} = Pr\left\{|\delta r| \le k\sigma_r\right\} Pr\left\{|\delta\alpha| \le k\sigma_\alpha\right\} = 4\Phi_0^2(k).$$

The lowest guaranteed P_{G^*} is:

$$P_G \ge P_G^\rho\left(\frac{\delta\rho}{2\sigma_r}\right) P_G^\theta\left(\frac{\delta\theta}{2\sigma_\alpha}\right),$$

$$P_G^\rho\left(\frac{\delta\rho}{2\sigma_r}\right) \ge \Phi_0\left(k - \frac{\delta\rho}{2\sigma_r}\right) + \Phi_0\left(k + \frac{\delta\rho}{2\sigma_r}\right),$$

$$P_G^\theta\left(\frac{\delta\theta}{2\sigma_\alpha}\right) = \Phi_0\left(k - \frac{\delta\theta}{2\sigma_\alpha}\right) + \Phi_0\left(k + \frac{\delta\theta}{2\sigma_\alpha}\right).$$

To choose detection threshold M, the probabilities P_{RTD} and P_{FTD} are introduced. The first one is determined as a *probability to obtain exactly M measurements from N consecutive scans*:

$$P_{RTD}(M,N) = \binom{N}{M}\left(P_G P_D^{HT}\right)^M \left(1 - P_G P_D^{HT}\right)^{N-M}.$$

The second probability is determined as a *probability to obtain at least one FA per scan in the considered strip in exactly M scans from N consecutive scans*:

$$P_{FTD}(M,N) = \binom{N}{M}\left[1 - \left(1 - P_{fa}^{HT}\right)^\mu\right]^M \left(1 - P_{fa}^{HT}\right)^{\mu(N-M)},$$

where P_D and P_{fa} are considered constant and $\mu = \mu(\rho, \theta, \Delta\rho)$ is the number of elementary volumes in the strip. A suitable detection threshold is chosen to maximize P_{RTD} at fixed P_{FTD}.

An additional *velocity selection* of measurements in each detected track (ρ_m, θ_l) is performed to filter the remaining FA. Let $N_{m,l}$ measurements are associated with this track. The velocities corresponding to each possible measurement pair are computed:

$$v_{i,j} = \left\{ \frac{r_i \cos(\theta_l - \alpha_i) - r_j \cos(\theta_l - \alpha_j)}{\delta V (T_i - T_j)} \right\}_{Int} ,$$

where $i, j = \overline{1, N_{m,l}}$, $T_i \neq T_j$ - moments of measurements arrival, δV - velocity discretization step. If $\left| v_{i,j} \right| \in \left[v_{min}, v_{max} \right]$, it votes in a set of $n_v = 2\frac{v_{max} - v_{min}}{\delta V}$ accumulators. The HT track detection is confirmed when the number of measurements in any of velocity accumulators exceeds threshold M.

3. HT track detector implemented in MHT

The standard MHT measurement oriented version is described in [2]. The track initiation procedure takes place in following cases:

- *Case 1* (C1): at the first scan;
- *Case 2* (C2): when a measurement does not fall in any gate of existing tracks;
- *Case 3* (C3): when MHT considers every measurement in a gate of each track as a *potential* track.

A HT track detector (initiating rectilinear trajectories) is proposed to filter the FA in cases C1 and C2 before the application of the standard MHT tracks initiation procedure. The application of this procedure in case C3 is a source of redundant tracks, but here it is left unchanged as an effective tool for recognition and resolution of closely spaced tracks.

A description of the proposed algorithm is given in Fig. 2 (the standard MHT steps are written in italic, the new steps are written in bold). The algorithm starts with a HTA measurement accumulation. If a track is detected all measurements accumulated in the corresponding accumulator during the last N scans are processed scan-by-scan, by second standard MHT algorithm. Its purpose is to initiate and evaluate a new standard MHT cluster containing MHT tracks and hypotheses in it. This procedure is performed in the remaining time of the current scan frame. Because of this second MHT algorithm (used in parallel), the resulting MHT algorithm version is denoted here *MHT ^{2}-HT*. Finally, the new cluster is added to the others and starting at the next scan it is processed in standard way.

First scan:

Read basic input data & first scan data;

-HT initiation & observation accumulation;

Next N-1 scans:

Read the next scan data

-HT observation accumulation;

Next scans after the first N ones:

-Read the next scan data,

-Delete old observations associated to tracks form HT accumulators;

-HT observations accumulation;

-HT track detection & velocity selection;

---If an observation falls in the track gate of this cluster & in a track gate from another one:

----Make super cluster from these clusters.

---If an observation is out of all gates, then make a new cluster;

-For each old cluster:

--For each observation in the cluster:

---For each hypothesis in the cluster:

----For each track from the hypothesis:

-----If the observation falls in the track gate:

------If this track-observation pair is not encountered yet:

-------Create a new track from this pair;

------Add a new hypothesis with a new track in the cluster;

----Make a new track from the current observation;

----Add the new track to the above hypothesis;

---Leave first M1 hypotheses & prune others;

--Combine tracks (closely spaced or made up of one and the same observations);

--Combine hypotheses made up of one and the same tracks;

--Leave best M2 hypotheses & prune others;

--Filter all tracks in the cluster;

--If a HT track is detected:

---use standard MHT to create a new cluster of tracks & hypotheses;

---add this cluster to already existing ones;

--Split clusters if possible;

-Delete clusters with no tracks in them.

Figure 2: The MHT^2-HT algorithm version

3. Measures of performance

A variety of measures of performance are formulated for MHT algorithm performance evaluation.[2] To estimate the noise resistance of the compared algorithms, just measures of performance depending on the clutter density P_{fa} are considered below. They are computed on the basis of the Monte Carlo simulation at scenario consisting of L independent runs. Within an experiment, for a given performance parameter C, the sample mean $\overline{\mu}_C^l$ over L runs is recursively computed.[2]

The following measures of performance[2,6] are computed, at each scan k, for the experimental data gathered for each MHT cluster, from its best hypothesis:

- *Expected number of tracks* N_{tr} - sample mean over L runs of the number of tracks $N_{tr\,k}$ ($N_{tr\,k}$ - the number of *Tentative* and *Confirmed* tracks at scan k).
- *Expected number of deleted tracks* N_{dt}: sample mean over L runs of the difference $N_{tr_{k-1}} - N_{tr_k}$. If $N_{tr_{k-1}} < N_{tr_k}$, it is set 0.
- *Expected number of false tracks* N_{ft} - sample mean over L runs of the subtraction $N_{tr\,k} - N_{tk}$ (N_{tk} - the number of targets in track at scan k).
- *Probability of at least* N *confirmed tracks without later deletion* P_{ndt}^N: a sample mean of the number of the occurrences of the event $\{N_{ndt} \geq N\}$ in L runs (N_{ndt} is the number of *confirmed* tracks existing till the run end).

4. Performance evaluation

4.1. Algorithms parameters

A standard Extended Kalman Filter is used in both MHT versions. It is based on the nonlinear model:

$$X_{k+1/k} = X_k + Tv_k \sin\psi_k ;$$
$$Y_{k+1/k} = Y_k + Tv_k \cos\psi_k ;$$
$$\psi_{k+1/k} = \psi_k ;$$
$$v_{k+1/k} = v_k ,$$

where the state vector $x' = (X, Y, \psi, v)$ consists of target coordinates, heading and velocity. The initial values X_0, Y_0, ψ_0 and v_0 are known. The radar sampling interval is T. No process noise is considered.

The measurement equation is:

$z_k = h(x_k) + v_k$, where: $z' = (r, \alpha)$ is the measurement vector,

$h(x_k) = \begin{pmatrix} \sqrt{X_k^2 + Y_k^2} \\ arctan\frac{y}{x} + \kappa\frac{\pi}{2} \end{pmatrix}$ is the measurement matrix, and $v_k \sim (0, R)$ is a white Gaussian measurement noise with covariance matrix $R = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\alpha^2 \end{pmatrix}$.

The probability of a new target appearance in an elementary volume, the detection probability of appearance of FA in an elementary volume are chosen equal for both algorithms:

$$P_{New\ t\arg et}^{MHT} = P_{New\ t\arg et}^{MHT^2-HT} = 4\cdot 10^{-5},\ P_D^{MHT} = P_D^{MHT^2-HT} = P_D^{HT} = 0.9,$$

$$P_{fa}^{MHT} = P_{fa}^{HT} = 10^{-5}.$$

It is also set for both algorithms: gate size - 16; number of hypotheses retained after each observation *M1* = 8; number of hypotheses retained after each scan *M2* = 4; expected track length - 60 scans. In MHT^2-HT it is also chosen $N = 6$ and $M = 4$. The number of HTA accumulators is chosen $N_\rho \times N_\theta = 27 \times 140 = 3780$. The velocity selection is performed in 8 accumulators at velocity bounds: $v_{min} = 0\ m/s$ and $v_{max} = 20\ m/s$.

4.2. Simulation

Results from $L = 100$ Monte Carlo independent runs are obtained from common simulation model and scenarios. Each run lasts 35 scans. The scenario includes two closely spaced ships rectilinearly moving on crossing trajectories:

Ship	X_0 [*km*]	Y_0 [*km*]	ψ [^{0}]	v_0 [*m/s*]
1	3	4	35	16
2	2	4	45	16

The measurement errors are modeled as Gaussian distributed zero-mean random variables with covariance $\sigma_r = 100m$ and $\sigma_\alpha = 0.3^o$ .The measurement misses are modeled with: $P_D = 0.8$. The number of FA is modeled as random variable with binomial distribution along the r axis, depending on P_{fa} and on the sizes of the elementary volume ($\Delta r = 100\,m,\ \Delta\alpha = 1^o$). Two scenarios with different flows of FA (moderate - $P_{fa}^{low} = 5\cdot 10^{-5}$ and dense - $P_{fa}^{high} = 1\cdot 10^{-3}$) with uniformly distributed coordinates are considered. The surveillance region which size is $r \in [0{,}16\ km],\ \alpha \in [0{,}90\ ^o]$ contains 160x90=14400 elementary volumes. The sampling interval is chosen $T = 10s$.

4.3. Simulation Results

The measures of performance obtained for the standard MHT at P_{fa}^{low} (denoted by “1”) and P_{fa}^{high} (denoted by “2”) and their values for the newly proposed MHT^2-HT algorithm at P_{fa}^{high} (denoted by “3”) are presented by Fig. 3 $\div$ 6. They illustrate the superiority of the proposed new algorithm:

- N_{tr}, N_{ft}, N_{dt}: While P_{fa}^{high} generally deteriorates the performance of the standard MHT algorithm, the MHT^2-HT algorithm shows a remarkable noise resistance - its plots obtained for P_{fa}^{high} coincide with these obtained by the standard MHT algorithm for P_{fa}^{low}.

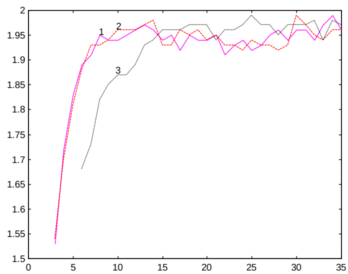

Figure 3: Average Target Number

$$N_t = N_t\left(P_{fa}\right)$$

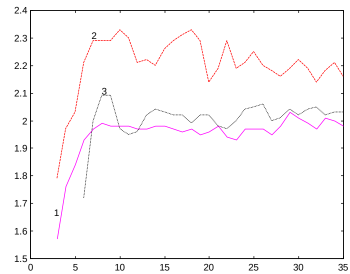

Figure 4: Average Track Number

$$N_{tr} = N_{tr}\left(P_{fa}\right)$$

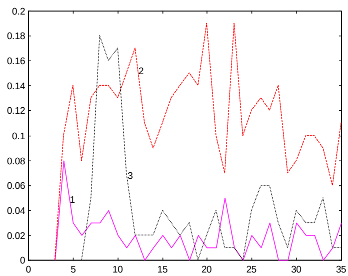

Figure 5: Average Deleted Track Number

$$N_{dt} = N_{dt}\left(P_{fa}\right)$$

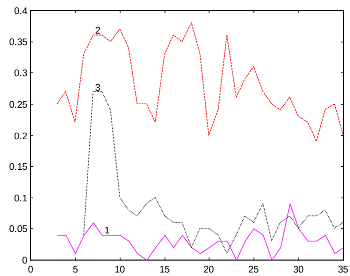

Figure 6: Average False Track Number

$$N_{ft} = N_{ft}\left(P_{fa}\right)$$

- P_{ndt}^{N}: The standard MHT algorithm shows increased “noise“ probability P_{ndt}^{3} at $P_{fa} = P_{fa}^{high}$ due to the increased number of false tracks, while the competing MHT^2-HT algorithm considerably reduces this probability: $P_{ndt}^{3}\left(MHT^2 - HT, P_{fa}^{high}\right) \approx P_{ndt}^{3}\left(MHT, P_{fa}^{low}\right)$.

Both algorithms provide $P_{ndt}^{i} \approx 1,\ i = 1{,}2$ and $P_{ndt}^{4} = 0$, for all considered P_{fa}.

5. Conclusion

A new version of the standard MHT measurement oriented algorithm is proposed and evaluated in the paper. A Hough Transform track detector is implemented in MHT to filter arriving false alarms. The measurements included in such tracks are arranged in MHT tracks by second, standard MHT algorithm used in parallel. The new MHT^2-HT algorithm shows a remarkable performance and noise resistance at the cost of delayed track detection procedure.

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